THE First (and last?) PROBLEM of EES 359

Calculate the mean Temperature of an earth with no albedo and no atmosphere (the essential blackbody radiation temperature). The governing concept is radiative equilibrium $E_{in} = E_{out}$, where E_{in} comes from the sun and E_{out} is the terrestrial radiation. Commonly this is done by averaging the heat input from the sun ($S=1367$ W/m^2) over the earth sphere, where $S^* = S/4$ or 342 W/m^2. This is then put into the Boltzman equation where $E = \sigma T^4$, where T equals the blackbody radiation temperature of the illuminated earth. The constant $\sigma = 5.67\ 10^{-8}$ W/m^2 K^4 , which would give us a radiation T of 278 K; with 30 % albedo that becomes the familiar 255 K (or -18 ^{o}C). In this case we average the solar input over the globe (S(local) is high at the equator because of orthogonal incidence and low at high latitudes because of 'slanted' incidence) and put that mean input into the Boltzman equation to get a mean temperature. The alternative is to calculate the local illumination values by latitude (S(latitude) or S_L) and then calculate local temperatures for each of these illumination intensities and take the average of those temperatures. This should give us another temperature value than the mean energy input / output approach. You can calculate these things through calculus or using little steps where you keep everything constant inside the step and add these up in a spreadsheet. With calculus we use the variation in one variable as a function of another as a continuous function, no little steps, or the steps are infinitesimally small. Some of you may go that way, but I treat here the layout of the numerical approach in Excel.

So here we go:

Take the earth as a perfect sphere with radius 6378 km ($=R_E$). You have to choose an appropriate step size for the latitude parameter, and one degree latitude steps may be OK as a first try (about 111. 3 km in N-S sense). To be exact, if you want to approximate between 0 and 1 ^{o}N, you should use the values at 0.5 ^{o}N as an average.

Now you want to calculate the amount of energy that is spread out over a 'cylindrical' surface (truly a double truncated cone surface) during a 24 hour rotation period of latitudinal distance ΔL (=111.3 km) and a radius that starts at R_E in the tropics but decreases with latitude ($=R^*$) to go to 0 at the pole– see drawing. Calculate how much energy is intercepted at each latitude, taking into account the angle of incidence and magnitude of R* and then the 'spread out' effect over the cylindrical surface that is rotating. Once you have the energy input in W/m^2 (average over 24 hours, no seasons), put it into the Boltzman equation and get the local latitudinal blackbody temperature. Multiply these temperatures with the surface area of the band, and you get the contribution of that latitudinal band to the mean global temperature. Sum all of these latitudinal band temperatures times surface areas up, multiply by 2 (for N and S hemisphere) and divide by the surface area of the earth ($4\pi\ R_E^2$) and you have found the mean radiation temperature of the earth.