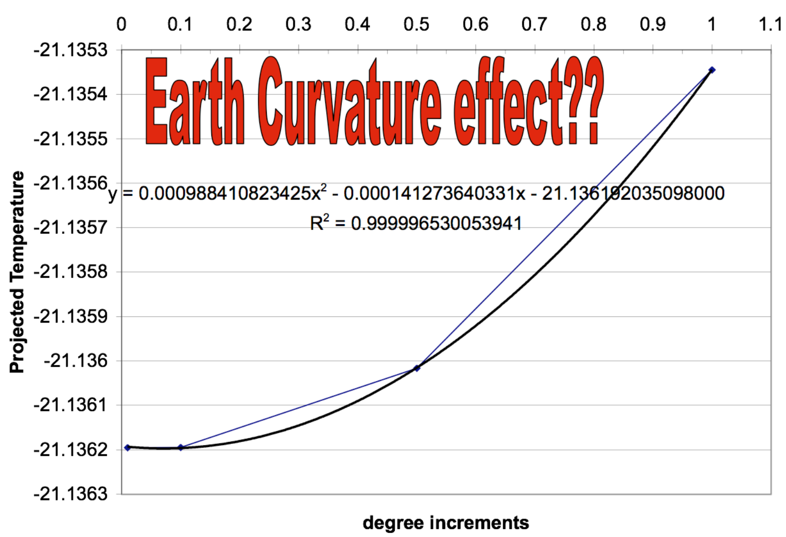
Answer to problem set 1 – EES 359

We can solve the problem first for 1 degree latitude intervals at zero albedo. The "mean energy input' solution equals $\{(1367/4) / \sigma \}^{0.25} = 5.48\ ^{o}C$. The 'equilibrium temperature' at 30% albedo is the familiar -18 ^{o}C. We enter in Excel the following columns: Albedo (fraction 0-1), latitude (degrees), latitude (radians), solar constant, S(lat), S*(lat), T(lat), R*, stripsurface (lat)*T(lat), as shown below for a 1 o latitude step simulation

albedo	Latitude	radians	S	Slat	S*	Tlat	R*	T*SS
0.3	0.00000001	1.74533E-10	956.9	957	305	271	6378	1207644201
0.3	1.00000001	0.017453293	956.9	957	304	271	6376	1207184394
0.3	2.00000001	0.034906585	956.9	956	304	271	6372	1206265023
0.3	3.00000001	0.052359878	956.9	955	304	271	6366	1204886581
0.3	4.00000001	0.06981317	956.9	954	304	271	6358	1203049801
0.3	5.00000001	0.087266463	956.9	952	303	270	6349	1200755663

The latitude column starts at a number close to 0 and then adds the lat step of 1 degree. When it calculates cos (lat) it uses the median value between two rows, so in row 1 it calculates the cos ($0+0.5\ ^{o}$); the cos value must be entered in radians, however. In the T calculation it uses Boltzman and local radiative equilibrium (IN=OUT) where IN is the spread-out solar flux averaged over 24 hours and no seasons. The R* value is the projected radius at a given latitude angle, again using the lower latitude value + a half step. The last column is the product of strip surface area and the local temperature. The whole column is then summed from $0 – 90^{o}$, and the sum is multiplied by two (to go from a half earth to the whole earth) and then divided by the surface of the earth; then deduct 273.15 to go from Kelvin to ^{o}C. You can repeat this calculation with smaller latitude steps and look at the difference in result. You then plot the results against the latitude step, fit a polynomial to these data and extrapolate to an infinitesimal small step, and you have the final answer. The results are

At 0 albedo, the 'mean energy input' result is **5.48 ^{o}C**, our calculated result is **2.37 ^{o}C**.
At 30% albedo, we obtain **-18.3 ^{o}C** from the mean energy input equation and **-21.1 ^{o}C** from our numerical approach. We thus end up with slightly lower blackbody temperatures. The 30% modern albedo also includes cloud albedo, so formally speaking, we should not use that 30% value for a planet without atmosphere!



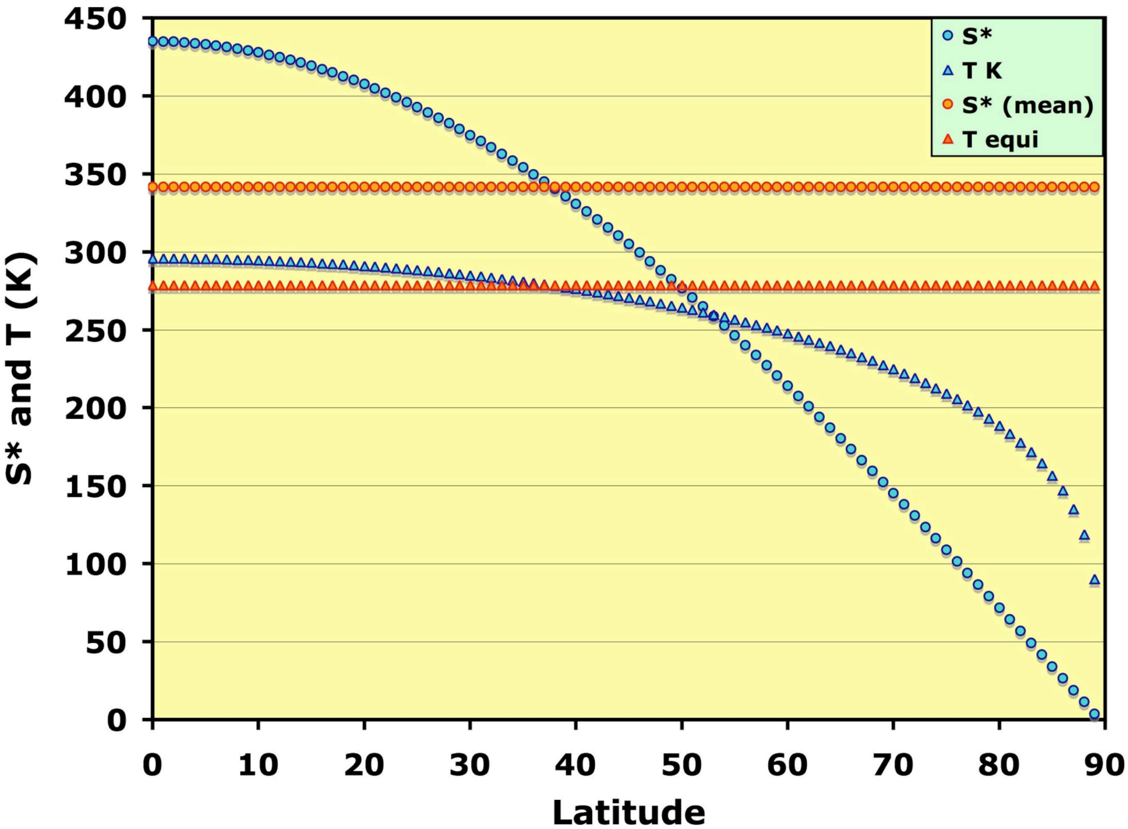


Figure 2. Results of the zero albedo calculations: S* is the calculated energy input per latitude band and T (in K) is the calculated temperature per latitude band. The S*(mean) is the S/4 enrgy input and T equi the associated black body temperature (278 K).

I took the calculated energy input values per latitude band (S*) with latitude steps of 0.01 degree, and treated them the same way as we do when calculating the average for the temperatures (calculate their average based on surface area weighting per latitude band). We then obtain the same 342 W/m^2 that we find by using simply S/4. This indicates that our overall treatment is right!